



XXXI. *Rules and Examples for limiting the Cases in which the Rays of refracted Light may be reunited into a colourless Pencil: In a Letter from P. Murdoch, M. A. and F.R.S. to Robert Symmer, Esq; F.R.S. Jan. 3, 1763.*

Read June 2, 1763. } 1. **L**ET SO, a small pencil of the solar light, pass through the refracting medium ABCD [TAB. XIV. Fig. 1.] whose opposite surfaces, represented by AB, CD, are parallel planes: then the violet rays OV, will, in the second refraction into the air, emerge parallel to the red; for both will be parallel to the incident ray SO, and consequently to each other: that is, Vv will be parallel to Rr, as is plain from the common principles of Optics.

2. If the light after its emergence is received on a screen placed any where beyond RV, it will be tinged with violet on the side Vv, and with red towards Rr: and if the incident pencil SO is exceeding small, all the intermediate colours will be seen in the same order as when light is refracted by a prism.

But if the incident pencil is not very small; or if the luminous body from which the rays are transmitted through a small opening at O, has a considerable breadth, like that of the Sun's disk; then so many rays of every kind will mix towards the middle of the spectrum as to produce a pure white; but at the extremities Vv and Rr, it will still be tinged

ed with violet and red: for a violet ray from the uppermost point of the Sun's disk will be more refracted than the other can be; and a red ray from the lowest part of his disk, will be less refracted than any other.

3 If  $BC$ , the distance of the refracting surfaces, is increased or diminished;  $RV$ , the distance of the extreme rays, will be increased or diminished in the same proportion: and if  $RV$  approaches very near to the aperture  $O$ , the colours will become imperceptible.

4. To reunite these rays, we may place another medium of the same refractive power, and of the same thickness ( $bc = BC$ ) as in the figure; so as the rays  $Vv$ ,  $Rr$ , &c. may enter its surface  $cd$  at the same angle as they emerged at from  $CD$ , or as  $SO$  entered  $AB$ ; and after refraction at the point  $o$  of the surface  $ab$ , to which they converge, they will be reunited into  $os$  the continuation of  $SO$ , in a pencil every way like the incident pencil  $SO$ , excepting that the light will have been somewhat weakened in its passage through the media.

5. Other things remaining, let the thickness of the second medium be  $cp$ , less than  $cb$  or  $CB$ , the surface parallel to  $cd$  being  $pe$ ; and the emergent rays  $\omega\sigma$  will be indeed parallel to the incident as formerly, but the spectrum will fall below the place of the screen where  $SO$  or  $os$  would fall. It will likewise be coloured, as the rays were not yet united at the point  $o$ . If the thickness be greater than  $cb$ , the spectrum will fall above the line  $SO$   $os$ , and the violet and red, after their intersection in  $o$ , will have changed sides.

6. Other things remaining, suppose the refractive power of the medium  $ac$  to be increased, making the extreme rays to intersect before they reach the surface  $ab$ ; in that case, let the medium be turned round upon an axis perpendicular to the plane of refraction (represented by the plane of the figure) in the order of the letters  $a, b, c$ , so that the angle of incidence of the rays  $Vv, Rr$ , the line  $vr$ , and the angle  $vor$  may be continually decreasing till the intersection  $o$  falls into the side  $ab$ ; and the rays will emerge colourless and parallel to the incident pencil  $SO$ ; above, or below, or in the line  $SOos$ , according to the assumed place of the axis of revolution.

If, on the contrary, the refractive power of the medium  $ac$  be diminished, and, with it, the angle of convergence of the extreme rays; the point where they would intersect falling beyond the surface  $ab$ ; the medium must then revolve the contrary way, in the order  $c, b, a$ ; to bring the point of intersection to the surface  $ab$ . But if the refractive power be so small that even when  $cd$  becomes almost coincident with  $Vv$ , the point of intersection falls still beyond  $ba$ , in that case the rays cannot be made to emerge colourless, otherwise than by encreasing the depth of the medium till its surface passes through the point of intersection. And in like manner, when the refractive power of the second medium  $ac$  is greater than that of  $AC$ , making the rays to meet within the medium, as at  $q$  a point in the line  $pe$ ; we may, instead of turning the medium round on an axis, cut off the part  $pa$ , leaving the surface  $pe$  parallel to  $cd$ ; and the emergent light will be colourless.

From

From these few principles we may determine the phenomena of light transmitted through parallelepipeds that are contiguous to the air, their position and refractive powers being given. Or we may dispose them so that the emergent light shall, or shall not, be tinged with colours.

And we already see (what shall be more distinctly explained below) that if light be transmitted through whatever number of media ( $A, B, C$ , &c.) all the refractions may be corrected by the equal and contrary refractions of the same number of the same media ( $c, b, a$ ,) similar and similarly situated to the former; provided there is a medium  $Z$  interposed between the two series, thus;  $A, B, C, Z, c, b, a$ ; and that the rays in their passage through  $Z$ , are parallel to one another.

7. But to give the rays this parallelism in their passage through  $Z$ , and to explain the several phenomena of refracted light, we shall need the following

# L E M M A, a PROBLEM.

Given (in Fig. 2.)  $DCB$  the difference of two angles  $ACD, ACB$ , and the ratio of  $DI$  the sine of the greater to  $BH$  the sine of the lesser being likewise given, to find the angles.

For  $DF$ , the sine of the given difference, write  $s$ , and for its cosine  $CF$  write  $c$ ; for the lesser sine  $BH$ , the letter  $z$ , and let the given ratio of  $DI$  to  $BH$ , be that of  $m$  to  $n$ , the radius  $CB$  being unity.

Then, having drawn  $FG$  perpendicular to  $DI$ ; from the similar triangles in this figure, we shall have  
CB

CB : CH :: DF : DG, or  $1 : \sqrt{1-z^2} :: s : DG =$   
 $s \sqrt{1-z^2}$ ; and CB : BH :: CF : GI, or  $1 : z :: c :$   
 $GI = cz$ . But (by Hypoth.) DI : BH ::  $m : n$ ;  
 that is DG + GI, or  $s \sqrt{1-z^2} + cz : z :: m : n$ ;  
 which gives  $\sqrt{1-z^2} : z$ , or CH : BH, or  $1 : \text{tang.}$   
 ACB ::  $m - nc : ns$ ; that is,  $\text{tang. ACB} = \frac{ns}{m - nc}$ .

In words—multiply the fine of the given difference  
 by the least term of the given ratio for a dividend:  
 from the greater term subtract the product of the co-  
 sine of the difference and the lesser term for a divisor;  
 and the quotient shall be the tangent of the lesser an-  
 gle ACB.

Or, if you prefer a geometrical construction; In  
 the semidiameter CB produced take CM to CB as  
 DI to BH; and in the tangent to the circle at B,  
 make BL to BC, as DF to FM, and BCL shall  
 be the lesser angle sought.

Or you need only join DM and draw the semidia-  
 meter CA parallel to it.

8. But before we apply this solution, it may be  
 proper to give a table of the refractive powers of glaſs,  
 water and spirit of wine, whether contiguous to the  
 air, or perhaps the fluids contiguous to glaſs: these  
 being the substances in which experiments may be  
 most conveniently made: and it is also necessary to  
 know the limitations that arise from those several  
 powers.

I.

When light passes from air into glass, and the angle of incidence is next to  $90^\circ$ , whose sine is unity ;

The sine of the refraction of the red }  
 rays =  $\frac{5}{7}$  is .6493508 = sin. - - } 40° 29' 33,6  
 And of the violet  $\frac{5}{8}$  = .6410256 = sin. 39 52 6

III.

From Water into Glafs.

$$\begin{array}{rcl} \text{Sin. incid. : s. refr. of the red :: 1 : } & \left. \begin{array}{l} \text{\textdegree} \quad ' \quad '' \\ 59 \quad 44 \quad 20\frac{1}{2} \end{array} \right\} & \\ & \text{0,863739 = s.} & \\ \text{Of the violet :: 1 : 0,859966 = s.} & \left. \begin{array}{l} 59 \quad 18 \quad 45 \\ \hline 0 \quad 25 \quad 35\frac{1}{2} \end{array} \right\} & \end{array}$$

The difference of which ——— is the greatest divergence.

IV.

From Air into Spirit of Wine.

$$\begin{array}{rcl} \text{Sin. incid. : s. refr. of the red :: 1 : } & \left. \begin{array}{l} \text{\textdegree} \quad ' \quad '' \\ 47 \quad 10 \quad 20,2 \end{array} \right\} & \\ & \text{0,7334001 = s.} & \\ \text{Of the violet :: 1 : 0,7266366 = s.} & \left. \begin{array}{l} 46 \quad 36 \quad 18,6 \\ \hline 0 \quad 34 \quad 1,6 \end{array} \right\} & \end{array}$$

The difference of which ——— is the greatest divergence.

V.

From Spirit of Wine into Glafs.

$$\begin{array}{rcl} \text{Sin. incid. : s. refr. of the red :: 1 : } & \left. \begin{array}{l} \text{\textdegree} \quad ' \quad '' \\ 62 \quad 18 \quad 0,1 \end{array} \right\} & \\ & \text{0,8853964 = s.} & \\ \text{Of the violet :: 1 : 0,8821802 = s.} & \left. \begin{array}{l} 61 \quad 54 \quad 24 \\ \hline 0 \quad 23 \quad 36 \end{array} \right\} & \end{array}$$

And their difference ——— is the greatest divergence.



These numbers are partly transcribed from Sir Isaac Newton, and partly computed by a rule of Mr. Euler in the Philosophical Transactions.

They are indeed carried on to more decimal places than the experiments hitherto made can well bear: but it is hoped that hereafter methods may be devised to measure the refractions of light to a very great degree of precision.

9. When a slender pencil SO, is refracted by the surface of a denser medium OT (Fig. 3.) the extreme rays being OV, the violet, and OR the red; we have seen that the surface RVT, at which the rays pass again into the rarer medium, being parallel to the first surface OT, the extreme, and all the intermediate, rays will emerge parallel to each other, and to the pencil SO.

But if the last surface RVT cuts the former in a line perpendicular to the plane of refraction at the point T, on the side of the radiant point S, then the extreme rays being refracted at the points V, R, will converge to some point F in the rarer medium: and if the light be received on a screen at F, it will be colourless; if nearer to the refracting medium, or farther from it, it will be tinged, but on different sides.

Thus if the denser medium is water, and the surrounding medium is air; the angle of incidence LOS being  $20^\circ$ , the angle of divergence VOR will be  $7' 46''$ . And OVP the angle of incidence at the second refraction for the violet rays being taken of  $30^\circ$ , the angle of convergence RFY will be  $14' 26''$ .

On the contrary, if the plane VRt, (Fig. 4.) which terminates the denser medium cuts the first refracting plane

plane on the other side of the perpendicular OL, the rays will diverge from some point  $f$  on the other side of the second surface: the violet ray OV being more refracted from the perpendicular VP, than the red is from the perpendicular R $p$ .

And it is evident, that if the distance (OT or Ot) of the point of incidence from the edge of a prism, the angle of incidence LOS, and the angle of the prism (OTV or OtV) are given, together with the refractive powers of the media, the lines OV, OR, will be given in magnitude and position. And thence the distance VR being given, with the angles of refraction at the second surface, the points, F or  $f$ , to which the rays converge, or from which they diverge, will be given. And their locus, or the Curve in which all these points are found, may be assigned; whether the angle of the prism is constant, and the angle of incidence is variable, or the contrary; and whether the rays are refracted, or, at a certain obliquity, come to be reflected by the second plane.

10. If it is further required that the extreme, and all the intermediate, rays which meet at F (in Fig. 3.) should thenceforth remain united in a colourless pencil: through the point of convergence F draw (by the lemma) the line ZX, making the angles ZFR, ZFV, such that their difference RFV being the given angle of convergence, their sines may be as the sines of refraction of the red and violet rays, when they pass from a given denser medium GKH into the air, at a common angle of incidence: and HFG perpendicular to ZX will be the line in which the surface of that medium must cut the plane of refraction, when the rays RF, VF, are refracted into the  
same

same line FN. And if the medium be terminated on the other side by any plane KN to which FN is perpendicular, the pencil NY, continued in the air, will remain colourless.

For instance, if the medium GK is glass, and the angle RFV is  $14^{\circ} 26''$ , ZFR the angle of incidence of the red rays will be found of  $17^{\circ} 54' 14''$ ; and the angle of refraction XFN, common to all the rays, will be  $12^{\circ} 6' 34'' \frac{1}{2}$ .

But if the plane HG, to which ZX is perpendicular, passes not thro' the point of reunion F, but on this or the other side of it; the rays in their passage thro' the medium, though parallel to each other, will be laterally separated.

11. Let a ray SOL (Fig. 5.) of a mean degree of refrangibility be refracted by AB the side of a glass prism ABC, so that the refracted ray OM may be perpendicular to the side of the prism AC; it is required to apply to this another prism of a differently refracting substance, as of water, so that the ray Mo being refracted at o, by the side DC, the refracted ray so may be parallel to OS.

The angle of incidence SOP, and the refractive power of the glass being given, the angle SOM, and its supplement LOM, are given produce Mo to n; and because os is to be parallel to LO take for the difference of the angles in the lemma, the given angle nos ( $=$ LOM), and through the point o draw rop, so that the sine of pon may be the sine of pos, as the sine of incidence to that of refraction, when a meanly-refrangible ray passes from water into air; and DoC, perpendicular to rp, will be the position of the side required.

We have here supposed the ray  $SO$  to be homogeneous, of a mean refrangibility ; but if it is a ray from the Sun the image at  $s$  will be very much tinged. The colours will have been separated at  $O$  ; a small matter more at  $M$ , but they will diverge very considerably at  $o$  ; for setting aside the refractions at  $O$  and  $M$  ; that is, supposing a pencil  $Mo$  to pass unrefracted in water till it falls upon a surface of air at an angle of incidence of about  $47^{\circ} 32' \frac{1}{2}$ , the divergence of the extreme rays will be about  $2^{\circ} 51' \frac{1}{2}$  : a small difference of sines answering to a considerable difference of the angles when they approach to  $90^{\circ}$  : the ultimate difference to which they converge, being (from water into air)  $7^{\circ} 26' \frac{1}{4}$ .

12. Let a pencil of the solar light  $SO$  (Fig. 6.) fall upon the surface of water  $BC$ , the extreme rays being refracted into  $OV$ ,  $OR$  ; it is required to assign the glass prism  $PNn$  (whose section  $PNn$  is an isosceles triangle) such, that the base  $Nn$  being parallel to  $SO$ , and the surface of the water  $AC$  being inclined to the base  $Nn$  in the same angle as the surface  $BC$  ; the extreme rays, in their passage through the glass prism, shall be parallel ; and all the rays shall emerge colourless in the line  $SO os$  ; that is, in the incident ray produced thro' both the media.

The angle  $SOB$ , and the refractions from air into water, being given, the angles  $VON$ ,  $RON$ , and their difference  $VOR$ , are given. Draw therefore, by the lemma, the line  $OG$ , making the sine of  $ROG$  to that of  $VOG$ , as the sine of refraction of a red ray, in passing from glass into water, is to the sine of refraction of a violet ray, their angles of incidence being equal, and  $PN$  perpendicular to  $OG$  will be the intersection  
of

of the plane of refraction with the side of the prism that is required.

Thus the angle SOB being  $30^\circ$ , VOR will be  $18' 12''\frac{1}{2}$ , VOG =  $50^\circ 38' 4''\frac{1}{2}$ , ROG =  $50^\circ 19' 52''\frac{1}{2}$ . whose sines are as the sines of refraction of the violet and the red, in passing from glass into water at a common angle of incidence. And therefore, the angles of the emergence of the rays OV, OR, in passing from water into glass will be equal, that is Vv will in its passage through the glass prism, be parallel to Rr, and the rays meeting with equal and contrary refractions at the points v, r, o, as they suffered at V, R, O, will emerge colourless at o.

Yet we must not be surprized if the pencil os is not absolutely pure light (even supposing, the matter, the figure, and the disposition, of the media to be faultless) because (1°) perhaps the refractive powers have not been determined with sufficient exactness (2°). If the glass plate which contains the water be not very thin, the light will have received a slight tincture in passing through it at O: This however may be remedied by confining the water between two glass prisms. And (3°) it is scarce possible to make experiments of this kind with a pencil of light so slender as the theory prescribes (see § 2.)

But proper allowances being made on these accounts, and the refracting planes adjusted as the lemma directs, the light will emerge sufficiently pure to justify the theory. And the refractions of either medium being given, it will appear from the experiments whether those of the other medium have been determined with sufficient accuracy.

Observe

Observe likewise, that as, in practice, we must fit the water to the glass, not the glass to the water, we are to begin by assuming  $VR$  of a convenient magnitude; and supposing the rays  $Vv$ ,  $Rr$ , &c. to be parallel within the glass, find the point  $O$  to which they converge in the water, through which a plane may be drawn which shall send them out into the air, in a colourless pencil  $OS$ .

## R E M A R K S.

### I.

The 8th experiment in Sir Isaac Newton's optics (Book I. Part 2.) seems to have been made under the conditions which are limited by the foregoing problem; though he does not specify these conditions. For, it is to be presumed, he did not combine his prism and water at random, but adjusted them so as to produce the expected effect. It is observed likewise, that he does not give us a description of his experiment so particular as, in most instances, he was wont to do. He thought perhaps that the consequences he deduces from it might sufficiently explain his meaning; especially as he had, in the foregoing propositions, fully established the principles of his theory.

However this be, several persons of skill and address in optical matters, have produced experiments in contradiction to that of Sir Isaac, and have affixed meanings to his conclusions which he never could intend, without being grossly inconsistent with himself: an

imputation from which common candor and decency ought to have protected so great a name \*.

For instance, when he says that "light as often as by contrary refractions it is so corrected that it emergeth in lines parallel to those in which it was incident, continues ever after to be white"; can this assertion possibly bear the meaning they would obtrude upon us? Had Sir Isaac so entirely forgot his own doctrine as not to know, That if the glass prism  $PNn$ , in the last scheme, is, any where above  $Vv$ , terminated by a plane to which the pencil  $SO$  is perpendicular, the rays  $Vv$ ,  $Rr$ , &c. though emerging parallel to  $SO$ , will exhibit their several colours? The sense therefore which the experiments affix to Sir Isaac Newton's words being so absurd, had not they done better to look out for one that was consistent with his theory? and such a one they would have found by only drawing a figure like the foregoing; where the rays of the pencil, reunited in  $os$ , as well as when separated within the glass prism, are parallel to each other and to the incident pencil. But, if the water is terminated by a plane different from  $AC$ , passing through the point  $o$ , and making the rays (no longer parallel to  $SO$ ) to diverge, then the light will, by degrees, in passing on from  $o$ , become coloured: which is Sir Isaac's other position.

To this meaning his own words ought to have led the objectors. It was light, not separate rays, which

\* The reader ought to be told, that it is not here intended to detract from the merit of the late Mr. Dollond's improvement of refracting telescopes; but only to correct a mistake of his concerning that difference of dispersion of rays, which he has so happily applied to use.

emerged

emerged in his experiment ; and which (being parallel to the incident light) continued to be colourless.

He adds farther, “ the permanent whiteness argues, that in like incidence of the rays, there is no separation of the emerging rays”: as much as to say, that in his experiment (as in our 6th Figure) the pencil, in passing or repassing, is supposed to meet with surfaces of equal refractive powers, similarly situated.

The other cases in which refracted light may recover its whiteness, although it emerges not parallel to the incident, or may be tinged though parallel to it, Sir Isaac does not treat of: the experiment he had made, being sufficient for the purposes to which he applies it. But he assures his readers, that if they will argue truly upon his theory, trying all things with good instruments, and sufficient circumspection, the expected event will not be wanting. And the fact is, that in all the experiments which have been made, if none of the necessary data are wanting, the appearance of the emerging light may be certainly predicted.

## II.

When a slender pencil of light is refracted at the surface of any medium, the extreme rays, the violet and red, and the several intermediate rays, each of its particular degree of refrangibility, will all diverge from, or converge to, the same physical point: or when that point, by altering the position of the plane, is thrown to an infinite distance, will all of them become parallel. And it appears from the foregoing solution, that such parallelism may always be effected,



whatever be the refracting power of the medium  $PNn$ , provided that, in a given medium, the quantities  $m$ ,  $n$ , &c. of the lemma, which represent the sines of refraction of the several sorts of rays, to a common sine of incidence, continue to be in constant ratios to one another.

Conversely, if, from experiments such as that which Sir Isaac Newton made, it follows that, whatever be the refractive powers of the media, and the angle of incidence of the light, the pencils  $SO$ , *so*, may be made to reciprocate with each other, while all the sorts of rays, in passing or repassing through the prism  $PNn$ , become parallel; if, I say, this is confirmed by experiments, it is a proof that, for any given medium, the ratios of those quantities  $m$ ,  $n$ , &c. are invariable.

### III.

And hence Sir Isaac deduces the two theorems subjoined to his 8th experiment; by the first of which he contrives to make the ratios of the sines of refraction belonging to the several sorts of rays, to a common sine of incidence, when they pass from glass into air, to serve for finding the like ratios for the rays passing from water into air, without the trouble of new experiments.

His first theorem may be deduced in this manner:

Let all the sorts of rays, whether united in a pencil of light, or separated parallelwise by refraction, have the same angle of incidence whose sine is  $I$ , when they pass from a denser into a rarer medium; and let  $V$  and  $R$  stand for the sines of refraction of  
the

the extreme (or any two sorts of) rays. Then seeing by the experiments, the ratio of  $V$  to  $I$  is given, as also that of  $R$  to  $I$ ; the ratio of  $V-I$  to  $I$ , as also (invert.) that of  $I$  to  $R-I$ , and (exæquo) that of  $V-I$  to  $R-I$ , are given: for this last write the ratio of  $1$  to  $p$ .

In like manner, let the refractive power of the medium from which the rays emerge into the same medium as before, be increased or diminished, as also the common angle of incidence; and we need only write other marks  $v$  and  $r$  for the sines, and  $i$  for the common sine of incidence; for we shall have as before  $v-i$  to  $r-i$  in a given ratio; which call that of  $1$  to  $q$ . And, from these two, we have  $\frac{V-I}{v-i} = \frac{R-I}{r-i} \times \frac{q}{p}$ . But  $p$  is always nearly equal to  $q$ ; in the refractions from glass, and from water into air, their difference is less than  $\frac{1}{4000}$  part of either; we may therefore put the ratio  $\frac{V-I}{v-i}$  equal to  $\frac{R-I}{r-i}$ ; which is the first theorem.

And thence, if one difference  $R-i$  becomes equal to  $r-i$ , the other differences  $V-i$ , &c. will be respectively equal to  $v-i$ , &c. and the same set of differences may be made to serve for several media, provided the sines of incidence are taken in their due proportion.

Thus when red rays pass from glass into the air, we have  $I : R :: 50 : 77$  and  $R-i : I :: 27 : 50$ , and when they pass from water into air  $i : r-i :: 3 : 1$ , and therefore, as we are to make  $R-i$  every where equal to  $r-i$ , we get, ex æquo,  $i : I :: 81 : 50$ , as Sir Isaac Newton finds it.

VI.

But to explain this matter a little farther, and obviate some difficulties concerning it, I shall add the following

E X A M P L E S.

The refractive powers being as marked above, let red rays fall from glass into air at the angle of incidence  $20^\circ$ , the angle of refraction will be  $31^\circ 47'$ .

Again, let them fall from water into air at an angle of  $34^\circ 1'$ , making their angle of refraction  $48^\circ 5'$ .

And the difference of the sines of  $31^\circ 47'$ , and  $20^\circ$  will be precisely equal to the difference of the sines of  $48^\circ 5'$ , and  $34^\circ 1'$ .

At the same angles of incidence  $20^\circ$  and  $34^\circ 1'$ , let the violet rays fall from glass and water into air; and the angle of refraction from the glass, will be  $32^\circ 14'\frac{1}{2}$  nearly, and that from the water will be  $48^\circ 38'$  nearly. And the difference of the sines of  $32^\circ 14'\frac{1}{2}$  and  $20^\circ$  will equal the difference of the sines of  $48^\circ 38'$  and  $34^\circ 1'$ , within .000488, or less than  $\frac{1}{2000000}$ th part.

We see likewise that the red and violet rays diverged from the glass medium at an angle of  $27'\frac{1}{2}$ ; but from the water at an angle of  $33'$ ; making the difference of divergence in this example  $5'\frac{1}{2}$ ; that is  $\frac{1}{3}$  of the whole divergence of the red and violet rays when refracted from glass into air, at incid.  $20^\circ$ .

Whence

Whence it appears, that although the differences of fines above specified, or the excesses in Sir Isaac's theorem, may, in refractions from different media into the same rarer medium, be made equal, it does by no means follow, that the divergences of the several sorts of rays (or if you chuse to call it their dispersion) will be the same in the two refractions; for Sir Isaac's excesses  $27$ ,  $27\frac{1}{9}$ , &c. are the excesses of fines; not of angles, as some opticians seem to have misapprehended.

Again, let an unrefracted pencil of light fall from common glass into the air (Fig. 7.) at the incidence  $39^\circ$ , and the angles of refraction will be,

Of the violet	- - - -	$79^\circ$	$2'$	$2''$
Of the red	- - - -	$75^\circ$	$43'$	$55''$

And their difference -  $3^\circ 18' 7''$  is the divergence of the extreme rays.

And the angle of refraction of the mean ray is  $77^\circ 16' 19''$ .—By *mean ray* is understood the ray whose sine of refraction is a geometrical mean between the sines of refraction of the extreme rays, the common radius being unity.

Let now the same rays be refracted the contrary way by a surface of water WT, then, to make the mean emergent ray parallel to the incident pencil, its angle of incidence must be  $86^\circ 37'\frac{2}{3}$ : and the extreme rays will now converge at an angle of  $20\frac{1}{8}$  minutes, nearly.

Through the point of convergence  $o$ , draw (by the Lemma) a plane  $wt$ , to terminate the water, and unite all the rays into a colourless pencil  $os$ : and this  
emergent

emergent pencil will be found to make with a perpendicular to the terminating surface an angle of  $49^{\circ} 6' \frac{1}{2}$ , and will be inclined to the first incident pencil in an angle of 14 degrees, 28 minutes, 20 seconds. Nor is there any other plane besides this which will thus unite the rays. If planes parallel to it cut the rays any where but in their point of convergence, they will be parallel to each other, but exhibiting their several colours. And planes not parallel to it, will every where give a coloured image, excepting only when they pass through the point of convergence; but then the rays having cross'd at that point, will thenceforth diverge from one another, and give a coloured spectrum.

From all which it appears that light refracted thro' different media may emerge colourless, although its first direction be considerably altered. And that its mean direction may remain the same, though its extremities be sensibly tinged with colours. Positions which, I know not by what mishap, have been deemed paradoxes in Sir Isaac Newton's theory of light, but which are really the necessary consequences of it.

*Of Telescopical Object-Glasses giving an Image free from Colours. Fig. 8 and 9.*

If the extreme rays, the red and violet, after one or more refractions, diverge from points D and d, the distance of the point of divergence of the least refrangible from the lens, being greater than that of the most refrangible, such a semidiameter of the last spherical surface, from which they are to pass into the  
the

the air, may be assigned, as shall unite the extreme, and all the intermediate, rays in the same focus F; neglecting the aberration from the figure.

The R U L E is this;

For the distances of the points of divergence from the lens, write  $D$  the greater, and  $d$  the least; the semidiameter of any of the given surfaces being assumed for unity: And  $\frac{M}{r}$ ,  $\frac{m}{r}$  expressing the ratios of the sines of incidence and refraction of the violet and red rays out of air into the last medium whose surface is required: the semidiameter of that surface will be  $\frac{M-m \times D d}{M D - m d}$ ; as may be easily demonstrated from a theorem of Dr. Smith, in the remarks subjoined to his Optics.

Thus if the last medium is glass, the semidiameter of the surface from which the rays pass into the air, must be  $\frac{D d}{78 D - 77 d}$ , it being, in this case,  $\frac{M}{r} = \frac{78}{50}$ ,  $\frac{m}{r} = \frac{77}{50}$ .

### EXAMPLE I.

Let  $MpCNcM$  (Fig. 8.) be a double convex lens of water confined between the plano-concave  $MTLN$ , and the meniscus  $MKNcM$ , both of glass, and having the radii of their surfaces contiguous to the water, equal to each other, or to unity: and if a ray  $Sp$ , parallel to the common axis of the lenses, after being refracted by the aqueous lens, have its

extreme rays, the red and violet, divergent from the points D and *d*; the distance of F, the focus where all the rays can meet, will be 8.898: and when this happens, the exterior surface of the meniscus, that is, the surface represented by MPKN, will have its radius to that of the inner surface McN, as 139 to 154.

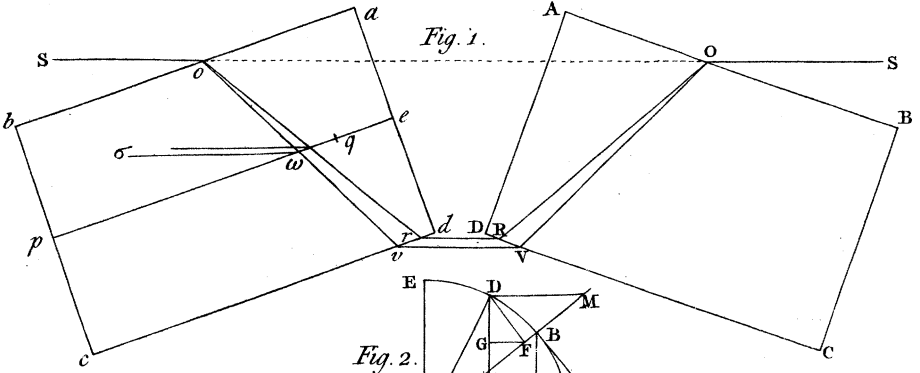
### EXAMPLE II.

When a double concave of glass (the radii of whose surfaces are unity) is inclosed in water, as in Fig. 9, the water being confined on one side by a thin glass plate TL, and on the other by a concentrick spherical shell MPKN; the semidiameter of this shell must be to unity as 471 to 547: and the focal distance CF, at which the colourless image is formed, will be  $4.77\frac{2}{3}$ . In these examples the thickness of the lenses is neglected; but it may easily be taken into the account, if it is thought necessary.

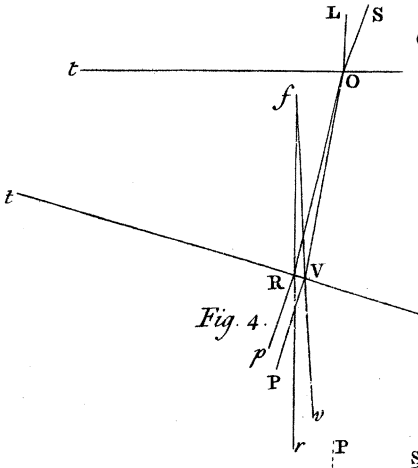
The same thing may be effected by means of any media of different refractive powers: for the semidiameter of the last refracting surface being determined according to the foregoing rule, the nearer distance of the points of divergence (*d*) of the more refrangible rays will be so compensated by their greater refrangibility, that all the rays will converge to the same focus F. And this without introducing any new principle into the science of optics, or any dispersion of light different from the refractions discovered by Sir Isaac Newton near a hundred years ago.

*Note The Figures Serving only for a general Illustration are not Geometrically Exact.*

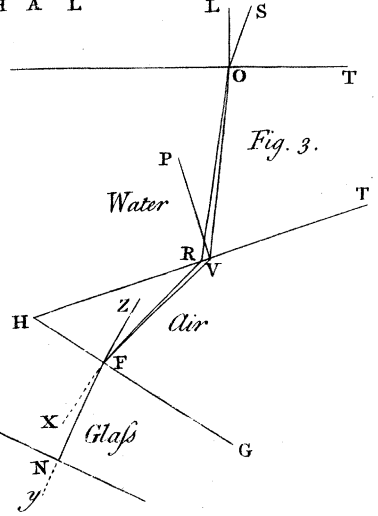
*Fig. 1.*



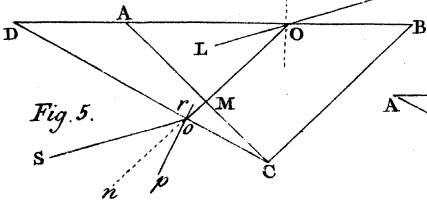
*Fig. 2.*



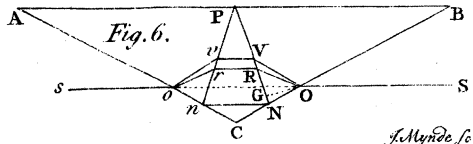
*Fig. 3.*



*Fig. 4.*



*Fig. 5.*



*J. Myrde sc.*